General equilibrium with substitutes and complements

Assume there are 2 consumers with the following utility functions:

$$u^A = 2x_1^A + x_2^A$$

$$u^B = \min\{x_1^B, 2x_2^B\}$$

The endowments are as follows:

$$\omega^A = (5, 10)$$

$$\omega^B = (10, 10)$$

Obtain the quantities and the price ratio at equilibrium.

Answer

Maximizing the utility for individual A, we have the following Marshallian demand:

$$x_1^A = \begin{cases} \frac{m^A}{p_1} & \text{if } \frac{p_1}{p_2} < 2 \\ [0, \frac{m^A}{p_1}] & \text{if } \frac{p_1}{p_2} = 2 \\ 0 & \text{if } \frac{p_1}{p_2} > 2 \end{cases}$$

$$x_2^A = \begin{cases} \frac{m^A}{p_2} & \text{if } \frac{p_1}{p_2} > 2 \\ [0, \frac{m^A}{p_2}] & \text{if } \frac{p_1}{p_2} = 2 \\ 0 & \text{if } \frac{p_1}{p_2} < 2 \end{cases}$$

For individual B:

$$x_1^B = \frac{2m^B}{2p_1 + p_2}$$

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Now, knowing that $m^A = p_1\omega_1^A + p_2\omega_2^A$, and since I am interested in finding the price ratio p_1/p_2 in this exercise, I normalize: $p_2 = 1$:

$$m^A = 5p_1 + 10$$

The same applies to the endowments of the other individual:

$$m^B = 10p_1 + 10$$

Thus, the Marshallian demands are:

$$x_1^A = \begin{cases} \frac{5p_1 + 10}{p_1} & \text{if } \frac{p_1}{p_2} < 2 \\ [0, \frac{5p_1 + 10}{p_1}] & \text{if } \frac{p_1}{p_2} = 2 \\ 0 & \text{if } \frac{p_1}{p_2} > 2 \end{cases}$$

$$x_2^A = \begin{cases} \frac{5p_1 + 10}{p_2} & \text{if } \frac{p_1}{p_2} > 2 \\ [0, \frac{5p_1 + 10}{p_2}] & \text{if } \frac{p_1}{p_2} = 2 \\ 0 & \text{if } \frac{p_1}{p_2} < 2 \end{cases}$$

For individual B:

$$x_1^B = \frac{2(10p_1 + 10)}{2p_1 + p_2}$$

$$x_2^B = \frac{(10p_1 + 10)}{2p_1 + p_2}$$

I analyze branch-wise; first, we check if $p_1/p_2 < 2$. That is, $p_1 < 2$. And verify with the conditions that the Marshallian demands are equal to the total quantities of the goods in the economy, starting with good 2:

$$x_2^A + x_2^B = \omega_2^A + \omega_2^B$$

$$0 + \frac{(10p_1 + 10)}{2p_1 + p_2} = 10 + 10$$

Solving for p_1:

$$p_1 = -10/30$$

This is absurd since prices cannot be negative.
We try the next branch: $p_1 > 2$

$$x_1^A + x_1^B = \omega_1^A + \omega_1^B$$

$$0 + \frac{2(10p_1 + 10)}{2p_1 + p_2} = 5 + 10$$

Solving for p_1:

$$p_1 = 1/2$$

This contradicts the fact that $p_1 > 2$. Then we try the last remaining branch: $p_1 = 2$. With this, we find:

$$x_1^B = 12$$

$$x_2^B = 6$$

We check the market conditions:

$$x_2^A + x_2^B = \omega_2^A + \omega_2^B$$

$$x_2^A + 6 = 10 + 10$$

$$x_2^A = 14$$

$$x_1^A + x_1^B = \omega_1^A + \omega_1^B$$

$$x_1^A + 12 = 5 + 10$$

$$x_1^A = 3$$

We have reached equilibrium since all market clearing conditions have been met.